Tasneem Batool 71143.

Exam II: MTH 213, Spring 2018

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(Bonus), (1 point). This is math 213 and my instructor name is Ayman Badawi . His office hours are on 4h oh. from to . We meet every, Sunday, Tuesday, and Thursday in Nab at am.

QUESTION 1. Imagine: then

(i) $O(\frac{x^{0.7}+3x^{0.2}-5}{x+7})$ equals to $= \frac{O(x^{0.7}+3x^{0.2}-5)}{O(x+7)} = \frac{x^{0.7}}{x} = x^{-0.3}$

4/4 (ii) $O(\sqrt[3]{x}+x)(x^2-x^{7/2}-2)$ equals to $= O(x^{1/3}+x) \cdot O(x^2 - x^{3.5} - 2)$
$= x \cdot x^{3.5} = x^{4.5}$

QUESTION 2. Imagine: The following Algorithm segment. Find the exact number of additions, multiplications, and subtractions that will be performed when the algorithm is executed. Then find the complexity of the Algorithm segment.

$m = 7; s = 9$
For $k := 3$ to (n^2+2)
$L = k*m + 2*s - 6$ 4 op.
For $i := 1$ to $(k+1)$
$s = s + m^3 + i - k^2$ 6 op.
s + m*m*m + i - k*k
next i
next k

No. of outer loop iterations = $L = n^2+2-3+1 = n^2$.

at $L = 1.(k=3)$
outer loop op : 4
inner loop op $= (6)(k+1-1+1)$
$= (6)(4) = 24.$
total op $= 4+24 = 28.$

at $L = n^2$ $(k = n^2+2)$
outer loop op = 4
inner loop op $= (6)(k+1)$
$= (6)(n^2+2+1)$
$= (6)(n^2+3)$
$= 6n^2+18.$
Total op: $4+6n^2+18$
$= 6n^2+22.$

$\therefore$ ~~Fo~~ Sum $= \frac{(28+6n^2+22)n^2}{2}$
$= \frac{(6n^2+50)n^2}{2}$
$= 3n^4 + 25n^2.$

Complexity:
$O(3n^4+25n^2) = n^4.$

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QUESTION 3. 1) Imagine: there are 220 flowers in a shop and each flower is either red or green or yellow (nothing else). You heard someone saying: There are at least 10 flowers that have the same color. You smiled and you told yourself : Definitely this person never took Math 213 or this person in the shop must be Ayman Badawi, since his arithmetic is very week. So can you make a better statement that is always true and no one can improve it? i.e., your statement: There are at least so and so...that have the same color.

4/4 Ans: $|\text{Domain}| = 220.$ $|\text{Co-domain}| = 3$ $m = \lceil \frac{220}{3} \rceil = 74.$

Result: There are at least 74 flowers of the same color.

2) Imagine: You travelled faster than the light and you discovered that only 727 persons were born between 2026-2031. Then there is a year where at most m persons were born in that year. Find the minimum value of m.
6 years

4/4 Ans: $|\text{Domain}| = 727.$ $|\text{Codomain}| = 6$ $m = \lfloor \frac{727}{6} \rfloor = 121.$

4) Use your own ENGLISH LANGUAGE : Explain to me the meaning of the answer in part (2).

4/4 Ans: By treating the number of people born as the domain, and number of birth years as co-domain, the answer in part (2) suggests that in ~~a~~ every possible scenario of this function, there will be a year (betw. 2026 and 2031), where no more than 121 people were born. This number ~~best~~ is the best assumption/conclusion as opposed to any numbers ~~greater~~ than it. greater.

Excellent

QUESTION 4. Imagine but fun: Given string1, say S_1 : 1 1 0 1 0 1 and string2, say S_2 : 0 0 1 1 1 1

a) Find $S_1 \wedge S_2$

$$\begin{array}{r} 110101 \\ \wedge\ 001111 \\ \hline 000101 \end{array}$$

$S_1 \wedge S_2 = 000101$

b) Find $S_1 \vee S_2$

$$\begin{array}{r} 110101 \\ \vee\ 001111 \\ \hline 111111 \end{array}$$

$S_1 \vee S_2 = 111111$

c) Find $S_1 \oplus S_2$

$$\begin{array}{r} 110101 \\ \oplus\ 001111 \\ \hline 111010 \end{array}$$

$S_1 \oplus S_2 = 111010$

d) Find $\neg S_1 \vee S_2$

$\neg S_1 = 001010$

$$\begin{array}{r} 001010 \\ \vee\ 001111 \\ \hline 001111 \end{array}$$

$\neg S_1 \vee S_2 = 001111$

QUESTION 5. Imagine I asked you to convince me (use T and F or 1 and 0) that $S_1 \to (S_2 \to S_3) \equiv S_2 \to (S_1 \to S_3)$ (Note $A \to B$ means if A, then B). let 0 for false and 1 for true.

S_1	S_2	S_3	$(S_2 \to S_3)$	$S_1 \to (S_2 \to S_3)$	$S_1 \to S_3$	$S_2 \to (S_1 \to S_3)$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	0	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	0	0	0	0
1	1	1	1	1	1	1

The outcomes on the truth table are same

hence $S_1 \to (S_2 \to S_3) \equiv S_2 \to (S_1 \to S_3)$

QUESTION 7. Imagine a set $A = \{1,2,3,4\}$. Define $\leq$ on $P(A)$ such that for every $a, b \in P(A)$, we have $a \leq b$ if $b \subseteq a$. We know that that $\leq$ is a partial order relation on $P(A)$ (note that $|P(A)| = 16$, do not show that. If possible find

(i) $\{2,3\} \wedge \{4\}$ $\quad \{2,3,4\}$
$c \leq \{2,3\}$
$c \leq \{4\}$

(ii) $\{1,3\} \vee \{3\}$ $\quad \{3\}$
$\{1,3\} \leq c$
$\{3\} \leq c$

(iii) $\{3\} \wedge \{2,4\}$ $\quad \{2,3,4\}$
$c \leq \{3\}$
$c \leq \{2,4\}$

(iv) $\{2,3,4\} \wedge \{1,2\}$ $\quad \{1,2,3,4\}$
$c \leq \{2,3,4\}$
$c \leq \{1,2\}$

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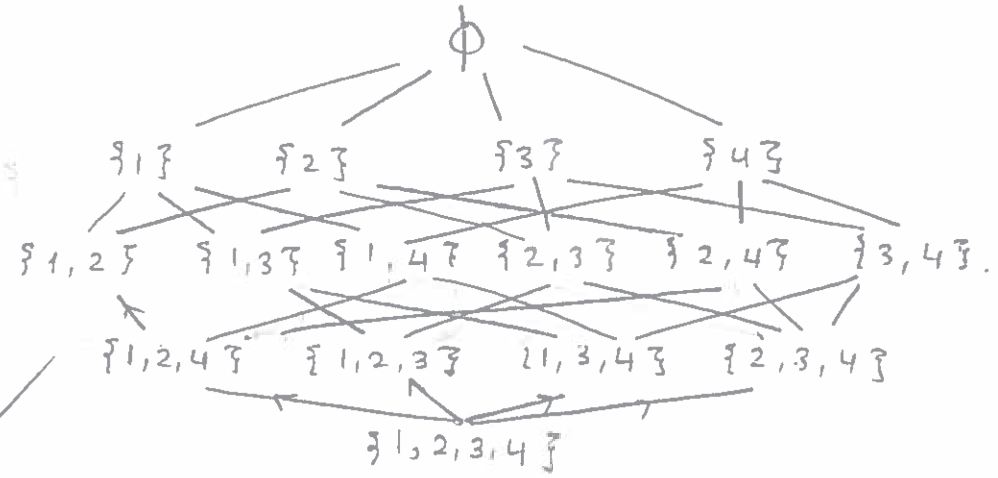
(v) Draw the Hasse diagram of $(P(A), \leq)$.

$P(A) = \{ \{1,2,3,4\}, \phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\} \}$



$2/2$

QUESTION 8. Imagine: Then 1) Convince me that $|(-6,4) \cup (4,10)| = |[-6,10]|$ (I do not think you need to use bijection functions HERE!!) $(-6,4) \cup (4,10)$ means all numbers between -6 and 4 and betw. $4, 10$.

$|(-6,4) \cup (4,10)|$ is infinite. ~~and~~ ~~________~~

$|(-6,4) \cup \{4\} \cup \{6\} \cup (4,10) \cup \{10\}| = |(-6,4) \cup (4,10)|$ → bc. adding finite elements to an infinite set keeps the set at infinite cardinality.

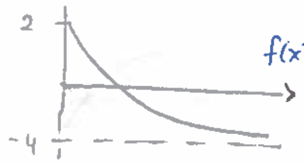
since ~~$\{6$~~ $(-6,4) \cup \{4\} \cup \{6\} \cup (4,10) \cup \{10\} = [-6,10]$

Hence, $|(-6,4) \cup (4,10)| = |[-6,10]|$

$1/1$

2) Convince me that $|[0,\infty)| = |(-4,2)|$ (use bijection functions somehow!!)

Construct bijective function $(0,\infty) \longrightarrow (-4,2)$



$f(x) = 6e^{-x} - 4$

$\lim_{x \to \infty} \frac{1}{e^x} = 0$

$\lim_{x \to 0} \frac{1}{e^x} = \frac{1}{1} = 1$

Hence

$|(0,\infty)| = |(-4,2)|$

$|(0,\infty) \cup \{0\}| = |(0,\infty)|$

Therefore $(0,\infty) \cup \{0\} =$ ~~$[0,\infty]$~~ $[0,\infty)$

$\Rightarrow$ ~~$[0,\infty]$~~

$|[0,\infty)| = |(-4,2)|$.

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3) If someone told you that a set D is uncountable? What does that mean?

Ans: This means you cannot construct a bijective function between set D and N (all positive integers) → so the cardinality of D is not equal to cardinality of N, and N is countable.

and D is not finite

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QUESTION 9. Imagine $A = \{-2, 2, -1, 1, 0, -7, 7, 14, 15\}$. Define $\equiv$ on A, where if $a, b \in A$, then $a \equiv b$ if $7 \mid (a - b)$ (in A) (i.e., a - b = 7c for some $c \in A$.)

a−b = 7m

(i) Convince me that $\equiv$ is an equivalence relation on A.

Ans: Axiom 1 : A−A. let a ∈ A.
"a = a" means a − a = 7m
0 = 7×0 , 0 ∈ A
Axiom 1 holds.
Axiom 2 : A − B. if "a = b", and a, b ∈ A, show "b = a"
let a = 0 b = 7
a − b = 0 − 7 = 7m
−7 = 7×−1, −1 ∈ A.
b − a = 7 − 0 = 7m
7 = 7×1, 1 ∈ A.
Hence "b = a"
Axiom 3 : A−B−C. let a, b, c ∈ A. if "a = b" and "b = c" then "a = c".
let a = 0 b = 7 , c = 14.
0 − 7 = −7 = 7×(−1), −1 ∈ A.
7 − 14 = −7 = 7×(−1) , −1 ∈ A
Add; 0 − 14 = −14 = 7×(−2) , −2 ∈ A.
∴ Axiom 3 holds and = is an equivalence relationship.

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OK but check 7 + 0 − 14

(ii) Find all equivalence classes of $(A, \equiv)$.

[0] = {0, −7, 7, 14}
[−1] = {−1}
[2] = {2}
[3] = {3}
[−2] = {−2}
[−3] = {−3}
[1] = {1, 15}

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(iii) view $\equiv$ as a subset of $A \times A$. How many elements does $\equiv$ have?

# of elements = $4^2 + 1 + 1 + 2^2 + 1 + 1 + 1$
= 23 + 1 + 1 = 25

QUESTION 10. Imagine $A = \{0, \{0, y\}, y, \{6\}, 6, x, \phi\}$, $B = \{\{0\}, \{\phi\}, \{6\}, \{6, x\}, 6, y, 23, 10, \{\{0\}, \{6, x\}\}\}$. Then Write T or F

(i) $\{\{0\}, \{6, x\}\} \in B$. T

(ii) $\{\{0\}, \{6, x\}\} \subset B$. T

(iii) $\{\phi\} \in A$ F

(iv) $\{\phi\} \in B$ T

(v) $\{\phi\} \subset B$ F

(vi) $\{\phi\} \subset A$ T

(vii) $\phi \in A$ T

(viii) $\{23, 10, y\} \in B$ F

(ix) $\{23, 10, y\} \subset B$ T

(x) $\{6\} \in A \cap B$ T

(xi) $\{6\} \subset A \cap B$. T

(xii) $(10, x) \in A \times B$ F

(xiii) $\{(\{0, y\}, 6), (y, \{0\})\} \subseteq A \times B$. T

(xiv) Find $A \cap B$ = {y, {6}, 6}

(xv) Find $B - A$. = {{0}, {φ}, {6, x}, 23, 10, {{0}, {6, x}}}

(xvi) Find $|A \times B|$. (i.e., find the cardinality of the cartesian product $A \times B$)

7 × 9 = 63.

Faculty information

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